Buckling Analysis of Anisotropic Sandwich Plates Faced with Fiber-Reinforced Plastics

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The small-deflection theory of orthotropic sandwich plates developed by Libove and Batdorf is extended to highly anisotropic sandwich plates with thin faces. Buckling coefficients of a simply supported rectangular symmetrical anisotropic sandwich plate under combined longitudinal compression and bending are evaluated using the Rayleigh-Ritz method. The results show that the multi-ply-faced orthotropic sandwich plate is stronger than a single-ply-faced anisotropic one and that the maximum longitudinal buckling strength occurs in plates with lower aspect ratios when the reinforcing fibers are longitudinal and in plates of higher aspect ratios when the fibers are oriented at about 40 deg with respect to the longitudinal axis.

Nomenclature

a = length of panel in x direction
b = width of panel in y direction
c = thickness of core
A_{mn},B_{mn},C_{mn} = undetermined coefficients, Eq. (9)
$D_{11},D_{12},D_{22},D_{23},D_{13},D_{23}$ = flexural stiffness of plate referred to geometric axes
D_1,D_2,D_3,D_4,D_5 = flexural stiffness ratios, Eqs. (12)
E_1,E_2 = elastic moduli referred to principal material directions
G_{12} = shear modulus referred to principal material directions
G_{xz},G_{yz} = transverse shear moduli of core in xz and yz planes, respectively
h_k,h_{k-1} = distances of the surfaces of kth layer as defined in Fig. 1c
J_x,J_y = shear rigidity ratios of core, Eqs. (12)
k_x = buckling coefficients, Eqs. (12)
m,n,p,q = half-wavelength integers
M_x,M_y,M_{xy} = internal moments per unit length
N_{0x},N_{0y} = maximum external compressive load per unit length in x and y directions, respectively
N_x = $N_{0x}(1-\gamma\cdot y/b)$
N = number of layers in each face
Q_x,Q_y = intensity of internal shears per unit length in xz and yz planes, respectively
$\bar{Q}^k_{ij}$ = transformed reduced stiffnesses of kth layer in a face
S_x,S_y = shear stiffnesses of core per unit length in xz and yz planes, respectively, Eq. (2)
t = thickness of face
w = deflection of the midplane of plate in z direction
x,y,z = Cartesian coordinates, Fig. 1
$,x$ and $,y$ = $\partial/\partial x$ and $\partial/\partial y$, respectively
X_{mn} = bB_{mn}/S_x
Y_{mn} = bC_{mn}/S_y
γ = bending load coefficients, equal to the ratio of the length of the loaded edge to the distance between extreme compression fiber and neutral axis
θ = orientation of fiber measured from x axis, deg
μ_{12},μ_{21} = Poisson's ratios referred to principal material directions
$1,2$ = principal material directions in xy plane, Fig. 1

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Introduction

THE complexity in the analysis of sandwich plates is due to the presence of two types of couplings resulting from the geometry and mechanical behavior of faces and core: 1) geometrical and material asymmetry of the faces with respect to the midplane of the core induces coupling between the bending and stretching actions, and 2) the generally orthotropic nature (equivalent to anisotropy as far as analysis is concerned) of the face and core materials introduces coupling between the stretching and shearing actions. In practice, the generally orthotropic nature of the sandwich results from the arbitrary orientation of the principal material axes of the face layers and core relative to the geometric axes. As the in-plane rigidities of the core are insignificant, its principal axes are laid parallel to the geometric axes for easy construction. For effective directional utility, the principal material directions of modern fiber-reinforced plastic (FRP) composites are laid arbitrarily. Hence, the total anisotropy of the sandwich results from the face materials. Typical examples of such plates are those with symmetric angle-ply faces (hereafter referred to as anisotropic sandwich plates).

Simplifying assumptions have been made to study the influence and significance of anisotropy on the structural response of FRP-faced sandwiches. One such assumption is to neglect the coupling terms in the constitutive relations and to treat the plates as effectively (specially) orthotropic (hereafter referred to as orthotropic sandwich plates). Libove and Batdorf[1] developed a general small-deflection theory of flat orthotropic sandwich plate that was later used by Robinson[2] to analyze buckling and bending in simply supported orthotropic sandwich panels. Edgewise compressive buckling curves for a simply supported panel having one face of glass-fabric laminate and one of an isotropic material were presented by Norris.[3] Kuenzi et al.[4] prepared an extensive list of curves for buckling coefficients summarizing the data developed by the U.S. Forest Product Laboratory. The curves given are of sandwich panels having one facing of either of two orthotropic materials and the other of an isotropic material, both facings of orthotropic materials, and both facings of isotropic materials, all with cores of orthotropic or isotropic material. The energy method and a differential equation approach were applied by Chang et al.[5] for the general stability analysis of orthotropic sandwich panels with four different boundary conditions. From the comparison of experimental results obtained by Nordby and Crisman[6] with the theoretical results of the Forest Product Laboratory,[4] it was concluded that the latter are conservative. Wempner and Baylor[7] developed a nonlinear theory of unbalanced sandwich plates taking into account the orthotropy of both the facings and the core.

Bending analyses of multilayer circular sandwich plates have been made by Stickney and Abdulhadi,[8] and of rectangular plate by Azar,[9] and Azar and Johnson.[11] Static and dynamic behavior or multilayer orthotropic sandwich plates has been analyzed by Chan and Cheung,[12] and their stability by Chan and Foo[13] using the finite strip method. Folie[10] derived the governing differential equations along with the corresponding boundary conditions for orthotropic sandwich plates by the variational principle of minimum potential energy and solved them by numerical integration to get the stresses and deflections of a clamped plate. Overall and local instability of FRP-faced sandwich columns were discussed by Ishai and Saggi,[14] Webber et al.,[15] and Gutierrez and Webber.[16] The effect of the coupling of stretching and bending actions in an unbalanced orthotropic sandwich plate with faces of cross-ply laminated panels was illustrated with the help from results on simply supported plates under lateral loads developed by Monforton and Ibrahim.[17] Kulkarni et al.[18] analyzed the displacement response of randomly excited simply supported orthotropic sandwich plates.

In references cited above, it is assumed that the principal material axes are parallel to the geometric axes. When the axes are arbitrarily oriented, the material behaves anisotropically. Analysis of sandwich plates with anisotropic faces is more difficult than with orthotropic faces. As a result, the literature contains few references about the analysis of anisotropic sandwich plates. Schmit and Monforton[19] suggested the discrete element method for the finite deflection analysis of sandwich plates and cylindrical shells with unbalanced anisotropic laminated faces. However, they applied the discrete element method to obtain the results on finite deflection and elastic postbuckling behavior of balanced orthotropic sandwich plates and cylindrical shells. A modified stiffness formulation for predicting the flexural deflections and stresses of unsymmetric anisotropic sandwich plates was proposed by Monforton and Ibrahim.[20] Monforton[21] concluded with the help of finite element method that the neglect of bending/membrane coupling in case of unbalanced FRP-faced sandwich beams underestimates the deflections and internal forces. Ibrahim et al.[24] extended the modified stiffness method of Ref. 20 to the dynamic analysis of sandwich plates with anisotropic unbalanced laminated faces and concluded that neglecting the coupling could result in appreciable errors, which could reach as high as 150% for the deflection results and 50% in the frequency prediction.

Kingsbury and Pavicic[22] suggested a perturbation solution of the equations of motion derived using the Hamilton principle and Reissner variational theorem. Using this method, Stroud and Kingsbury[23] studied stresses and deflections in anisotropic sandwich plates. Pearce and Webber did theoretical[25] and experimental[26] buckling studies of sandwich panels with laminated faces under uniform edgewise loads. Numerical results were evaluated for both the overall buckling and face plate wrinkling of panels having typical carbon fiber faces with laminations arranged to give effective orthotropic properties. The experimental values for overall buckling were found to be reasonably close to the theoretical results; however, the experimental wrinkling loads were higher than the theoretical values.

The cursory look at the extensive literature presented above gives bending and dynamic analyses of anisotropic sandwich plates. However, none deal with buckling analysis. The objectives of this paper are to extend the general small-deflection theory of orthotropic sandwich plates developed by Libove and Batdorf[1] to the buckling analysis of FRP-faced anisotropic sandwich plates for which no rigorous solution exists and to demonstrate the effect of stretching/shearing coupling on the stability of such plates.

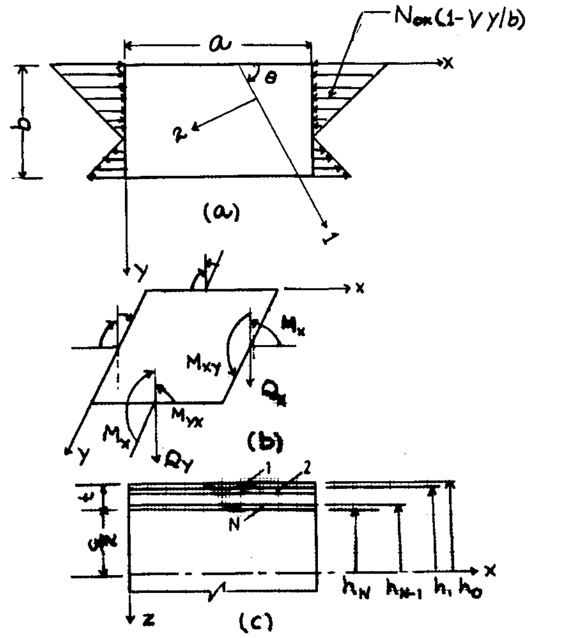

Fig. 1 Geometry and loading.

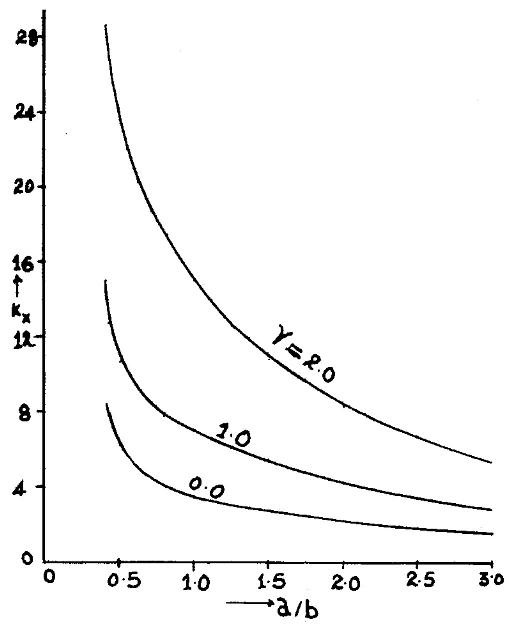

Fig. 2 Variation of k_x with a/b ($\theta=45$ deg, $c/t=20$).

Statement of the Problem

The present investigation is concerned with the buckling of elastic anisotropic sandwich panels with FRP facings. The faces are assumed to be balanced and thin compared to the core. In contrast to Ref. 25 where the laminations are arranged to give specially orthotropic faces, the number and orientations of the laminations are selected such that the faces behave highly anisotropically (i.e., generally orthotropically). The core is assumed to be thick, specially orthotropic, with infinite transverse normal rigidity, and quite flexible in the plane of the plate. Because of their thinness, the faces act as membranes. The edges parallel to y axis are subjected to linearly varying in-plane normal forces (i.e., combined longitudinal compression and bending). Figure 1 shows the geometry, external loading, and sign convention for the internal forces and moments.

Analysis

In case of balanced sandwich plates the in-plane deformations of the middle surface during buckling are zero. In the absence of in-plane deformations in the middle surface, the response of the plate can be characterized by transverse deflections and shear strains in the core. The distortions (i.e., curvatures) of an orthotropic sandwich plate are given in terms of lateral deflection w and transverse shear forces Q_x and Q_y by Libove and Batdorf.[1] To relate the couplings with the distortions, they defined seven constants consisting of five flexural stiffnesses and two transverse stiffnesses. To take care of the generally orthotropic nature of the FRP-faced sandwich plates in the present context, the five flexural stiffnesses of Ref. 1 are replaced by nine flexural stiffnesses. The modified form of force/deformation relations in matrix form can be written as

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = -\begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{Bmatrix} w,_{xx} - Q_{x,x}/S_x \\ w,_{yy} - Q_{y,y}/S_y \\ 2w,_{xy} - Q_{x,y}/S_x - Q_{y,x}/S_y \end{Bmatrix} \tag{1}$$

where M_x, M_y, and M_{xy} are the internal couples, S_x and S_y the transverse shear stiffnesses of core, and D_{ij} $(D_{ij} = D_{ji})$ $(i,j = 1,2,3)$ the elements of the symmetric bending stiffness matrix referred to geometric axes x and y. The transverse shear stiffnesses are given by

$$S_x = G_{xz}c, \qquad S_y = G_{yz}c \tag{2}$$

where G_{xz} and G_{yz} are the transverse shear moduli of the core in the xz and yz planes, respectively, and c the thickness of core. For evaluating D_{ij}, it is assumed that each layer is compressed to zero thickness and transferred to the middle surface of each face,

$$D_{ij} = \frac{(c+t)^2}{2} \sum_{k=1}^{N} \bar{Q}_{ij}^{k} (h_{k-1} - h_k) \tag{3}$$

where N is the number of layers in each face, $\bar{Q}_{ij}^k$ the transformed reduced stiffness of kth layer in a face, and h_k and h_{k-1} defined in Fig. 1c. The formula for evaluating the reduced stiffnesses Q_{ij} in terms of engineering elastic constants are[27]

$$Q_{11} = \frac{E_1}{1-\mu_{12}\mu_{21}} \qquad Q_{22} = \frac{E_2}{1-\mu_{12}\mu_{21}}$$

$$Q_{12} = \frac{\mu_{12}E_2}{1-\mu_{12}\mu_{21}} = \frac{\mu_{21}E_1}{1-\mu_{12}\mu_{21}}$$

$$Q_{33} = G_{12} \tag{4}$$

where E_1 and E_2 are elastic moduli, μ_{12} and μ_{21} Poisson ratios, and G_{12} the shear modulus referenced to the principal material axes 1 and 2. The transformed reduced stiffnesses $\bar{Q}_{ij}$ referenced to geometric axes x and y are

$$\bar{Q}_{11} = Q_{11}\cos^4\theta + 2(Q_{12} + 2Q_{33})\sin^2\theta\cos^2\theta + Q_{22}\sin^4\theta$$

$$\bar{Q}_{12} = (Q_{12} + Q_{22} - 4Q_{33})\sin^2\theta\cos^2\theta + Q_{12}(\sin^4\theta + \cos^4\theta)$$

$$\bar{Q}_{22} = Q_{11}\sin^4\theta + 2(Q_{12} + 2Q_{33})\sin^2\theta\cos^2\theta + Q_{22}\cos^4\theta$$

$$\bar{Q}_{13} = (Q_{11} - Q_{12} - 2Q_{33})\sin\theta\cos^3\theta + (Q_{12} - Q_{22} + 2Q_{33})\sin^3\theta\cos\theta$$

$$\bar{Q}_{23} = (Q_{11} - Q_{12} - 2Q_{33})\sin^3\theta\cos\theta + (Q_{12} - Q_{22} + 2Q_{33})\sin\theta\cos^3\theta$$

$$\bar{Q}_{33} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{33})\sin^2\theta\cos^2\theta + Q_{33}(\sin^4\theta + \cos^4\theta) \tag{5}$$

where θ is the orientation of the fiber measured from the x axis.

The Rayleigh-Ritz method is used for the determination of critical loads. The potential energy[1] V needed in the analysis consists of two parts: the strain energy due to the bending of the plate and the shearing of the core and the potential energy of the external forces,

$$V = -\frac{1}{2}\int_0^b\int_0^a \left[M_x\left(w,_{xx} - \frac{Q_{x,x}}{S_x}\right) + M_y\left(w,_{yy} - \frac{Q_{y,y}}{S_y}\right) + M_{xy}\left(2w,_{xy} - \frac{Q_{x,y}}{S_x} - \frac{Q_{y,x}}{S_y}\right)\right]\mathrm{d}x\mathrm{d}y + \frac{1}{2}\int_0^b\int_0^a\left(\frac{Q_x^2}{S_x} + \frac{Q_y^2}{S_y}\right)\mathrm{d}x\mathrm{d}y - \frac{1}{2}\int_0^b\int_0^a N_{0x}\left(1 - \gamma\frac{y}{b}\right)w_{,x}^2\mathrm{d}x\mathrm{d}y \tag{6}$$

where a and b are the length and breadth of plate, respectively. Substitution of Eq. (1) in Eq. (6) yields the final form of potential energy

$$V = \frac{1}{2}\int_0^b\int_0^a \left[D_{11}\left(w,_{xx} - \frac{Q_{x,x}}{S_x}\right)^2 + D_{22}\left(w,_{yy} - \frac{Q_{y,y}}{S_y}\right)^2 + D_{33}\left(2w,_{xy} - \frac{Q_{x,y}}{S_x} - \frac{Q_{y,x}}{S_y}\right)^2 + 2D_{12}\left(w,_{xx} - \frac{Q_{x,x}}{S_x}\right)\left(w,_{yy} - \frac{Q_{y,y}}{S_y}\right) + 2D_{23}\left(w,_{yy} - \frac{Q_{y,y}}{S_y}\right)\left(2w,_{xy} - \frac{Q_{x,y}}{S_x} - \frac{Q_{y,x}}{S_y}\right) + 2D_{13}\left(w,_{xx} - \frac{Q_{x,x}}{S_x}\right)\left(2w,_{xy} - \frac{Q_{x,y}}{S_x} - \frac{Q_{y,x}}{S_y}\right) + \frac{Q_x^2}{S_x} + \frac{Q_y^2}{S_y} - N_{0x}\left(1 - \gamma\frac{y}{b}\right)w_{,x}^2\right]\mathrm{d}x\mathrm{d}y \tag{7}$$

The edges of the plate are assumed to be simply supported with stiffeners such that they prevent shear deformation in the cross-sectional planes around the edges. The boundary conditions of such a plate are idealized as

$$w = M_x = Q_y = 0 \quad \text{at } x = 0 \text{ and } a$$

$$w = M_y = Q_x = 0 \quad \text{at } y = 0 \text{ and } b \tag{8}$$

The problem is formulated in terms of deflection w and internal shear forces Q_x and Q_y (proportional to shear strains). The Rayleigh-Ritz method of analysis needs the selection of values for those functions containing undetermined coefficients. These coefficients are then evaluated by minimizing the potential energy V of Eq. (7). In choosing suitable functions for w, Q_x, and Q_y, it is desirable that they inherently satisfy as many boundary conditions as possible, i.e., be independent of the values of the coefficients. Sometimes, in practice it is not possible to satisfy all of the boundary conditions because of the complexity introduced by the anisotropy of the material or by external constraints. The minimum requirement of the selected functions is that they at least satisfy the geometric boundary conditions, e.g., $w = Q_y = 0$ at $x = 0$ and a and $w = Q_x = 0$ at $y = 0$ and b in the present case. In addition, if they also satisfy the natural boundary conditions, e.g., $M_x = 0$ at $x = 0$ and a and $M_y = 0$ at $y = 0$ and b, the accuracy and convergence of the solution is improved. The functions chosen here are in the form of trigonometric series,

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin\alpha_m x \sin\beta_n y$$

$$Q_x = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} \cos\alpha_m x \sin\beta_n y$$

$$Q_y = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn} \sin\alpha_m x \cos\beta_n y \tag{9}$$

where m and n are half-wavelength integers; A_{mn}, B_{mn}, and C_{mn} the undetermined coefficients; and

$$\alpha_m = m\pi/a \quad \text{and} \quad \beta n = n\pi/b \tag{10}$$

The functions of Eq. (9) identically satisfy the geometric boundary conditions. The natural boundary conditions are not satisfied fully because of the presence of stretching/shearing coupling terms D_{13} and D_{23} in the matrix of Eq. (1). If the plate is isotropic or orthotropic, both the geometric and natural boundary conditions are identically satisfied. The shortcoming of not satisfying the natural boundary conditions in full can be compensated for by retaining a sufficiently large number of terms in the series of Eq. (9).

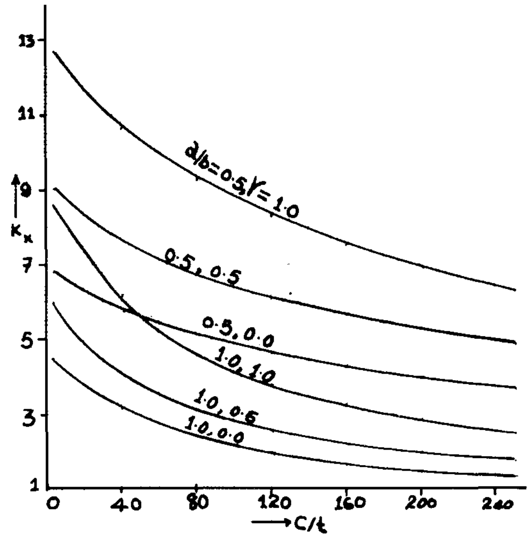

Fig. 3 Variation of k_x with c/t ($\theta = 45$ deg).

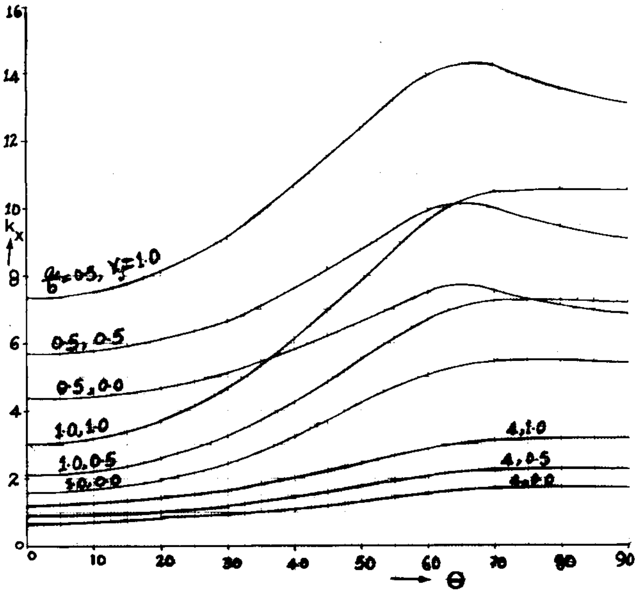

Fig. 4 Variation of k_x with θ ($c/t = 20$).

Table 1 Results for comparison with Ref. 25 N_{0x}, N/mm

	θ, deg								
	0	10	20	30	40	60	70	80	90
$N=4$ (present)									
N_{0x} (Ref. 25)	412.2	445.6	533.3	619.4	666.7	463.9	346.1	250.0	208.3
N_{0x}	423.8	462.9	553.5	641.4	685.6	469.4	349.0	253.6	213.8
D_4	0.0	0.0016	0.0033	0.0051	0.0069	0.0085	0.0050	0.0010	0.0
D_5	0.0	0.0001	0.0005	0.0017	0.0049	0.0025	0.0360	0.0256	0.0
$N=1$ (present)									
N_{0x}	423.8	428.7	443.5	459.5	468.7	356.6	285.6	233.6	213.8
D_4	0.0	0.1644	0.3350	0.5162	0.7049	0.8650	0.5086	0.0970	0.0
D_5	0.0	0.0061	0.0464	0.1748	0.4985	2.5539	3.6747	2.6104	0.0
$N=1$ and 4 (present)									
D_1	0.0184	0.0483	0.1442	0.3267	0.6331	1.6166	1.5822	0.7662	0.3151
D_2	0.0583	0.0630	0.0912	0.2021	0.5756	4.9477	10.9708	15.8766	17.154
D_3	0.0228	0.0529	0.1498	0.3342	0.6446	1.6537	1.6433	0.8405	0.3909
D_{11}, 10^{-6} N·mm	2.3967	2.2634	1.9023	1.4143	0.9235	0.2859	0.1734	0.1426	0.1397

Substituting Eq. (9) in Eq. (7) and integrating, one gets

$$V=\frac{ab}{8}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\Bigg[D_{11}\left(A_{mn}\alpha_m^2-\frac{B_{mn}\alpha_m}{S_x}\right)^2+D_{22}\left(A_{mn}\beta_n^2-\frac{C_{mn}\beta_n}{S_y}\right)^2+D_{33}\left(2A_{mn}\alpha_m\beta_n-\frac{B_{mn}\beta_n}{S_x}-\frac{C_{mn}\alpha_m}{S_y}\right)^2$$
$$+2D_{12}\left(A_{mn}\alpha_m^2-\frac{B_{mn}\alpha_m}{S_x}\right)\left(A_{mn}\beta_n^2-\frac{C_{mn}\beta_n}{S_y}\right)-\frac{32}{\pi^2}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\left\langle D_{23}\left(A_{mn}\beta_n^2-\frac{C_{mn}\beta_n}{S_y}\right)+D_{13}\left(A_{mn}\alpha_m^2-\frac{B_{mn}\alpha_m}{S_x}\right)\right\rangle$$
$$\times\left(2A_{pq}\alpha_p\beta_q-B_{pq}\frac{\beta_q}{S_x}-C_{pq}\frac{\alpha_p}{S_y}\right)\frac{mn}{(m^2-p^2)(n^2-q^2)}+\frac{B_{mn}^2}{S_x}+\frac{C_{mn}^2}{S_y}$$
$$-N_{0x}\alpha_m^2\left\langle A_{mn}^2-\frac{2\gamma}{b}A_{mn}\left(\frac{bA_{mn}}{4}-\frac{4b}{\pi^2}\sum_{q=1}^{\infty}\frac{nq}{(n^2-q^2)^2}A_{mq}\right)\right\rangle\Bigg] \quad (11)$$

where m, n, p, and q are to be chosen such that $m+p$ and $n+q$ are odd integers.

Minimization of potential energy V with respect to A_{mn}, B_{mn}, and C_{mn} gives three recurring simultaneous equations. Introducing the following notation, the three equations can be cast in nondimensional form:

$$J_x=S_xb^2/\pi^2D_{11},\quad J_y=S_yb^2/\pi^2D_{11},\quad K_x=N_{0x}b^2/\pi^2D_{11},\quad D_1=D_{12}/D_{11},\quad D_2=D_{22}/D_{11},\quad D_3=D_{33}/D_{11}$$
$$D_4=D_{13}/D_{11},\quad D_5=D_{23}/D_{11},\quad X_{mn}=bB_{mn}/S_x,\quad Y_{mn}=bC_{mn}/S_y,\quad \delta=a/b \quad (12)$$

The three recurring equations are

for $\dfrac{\partial V}{\partial A_{mn}}=0$:

$$\frac{\pi}{\delta}(m^4+D_2\delta^4n^4+4D_3\delta^2m^2n^2+2D_1\delta^2m^2n^2)A_{mn}-(m^3+2D_3\delta^2mn^2+D_1\delta^2mn^2)X_{mn}-(D_2\delta^3n^3+2D_3\delta m^2n+D_1\delta m^2n)Y_{mn}$$
$$-\frac{32}{\pi}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}(D_5\delta^2n^2+D_5\delta^2q^2+D_4m^2+D_4p^2)\frac{mnpqA_{pq}}{(m^2-p^2)(n^2-q^2)}+\frac{16}{\pi^2}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}(D_5\delta^3n^2+D_4\delta m^2+2D_4\delta p^2)$$
$$\times\frac{mnq}{(m^2-p^2)(n^2-q^2)}X_{pq}+\frac{16}{\pi^2}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}(D_5\delta^2n^2+2D_5\delta^2q^2+D_4m^2)\frac{mnp}{(m^2-p^2)(n^2-q^2)}Y_{pq}$$
$$-\pi\delta k_x(1-\gamma/2)m^2A_{mn}=\frac{8\delta k_{xy}}{\pi}\sum_{q=1}^{\infty}\frac{m^2nq}{(n^2-q^2)^2}A_{mq}=0 \quad (13)$$

for $\dfrac{\partial V}{\partial B_{mn}}=0$:

$$(m^3+2D_3\delta^2mn^2+D_1\delta^2mn^2)A_{mn}-\frac{1}{\pi}(\delta m^2+D_3\delta^3n^2+J_x\delta^3)Xmn-\frac{1}{2}(D_1+D_3)\delta^2mnY_{mn}-\frac{16}{\pi^2}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}(D_5\delta^3npq^3$$
$$+2D_4\delta m^2npq+D_4\delta np^3q)\frac{Apq}{(m^2-p^2)(n^2-q^2)}+\frac{16}{\pi^3}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}D_4(m^2+p^2)\frac{\delta^2nq}{(m^2-p^2)(n^2-q^2)}X_{pq}+\frac{16}{\pi^3}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}(D_5\delta^3npq^2$$
$$+D_4\delta m^2np)\frac{Y_{pq}}{(m^2-p^2)(n^2-q^2)}=0 \quad (14)$$

for $\dfrac{\partial V}{\partial C_{mn}}=0$:

$$(D_2\delta^3n^3+2D_3\delta m^2n+D_1\delta m^2n)A_{mn}-\frac{1}{\pi}(D_1+D_3)\delta^2mnY_{mn}-\frac{1}{\pi}(D_2\delta^3n^2+D_3\delta m^2+J_y\delta^3)Y_{mn}$$
$$-\frac{16}{\pi^2}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\times(2D_5\delta^2mn^2pq+D_5\delta^2mpq^3+D_4mp^3q)\frac{Apq}{(m^2-p^2)(n^2-q^2)}+\frac{16}{\pi^3}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}(D_5\delta^3mn^2q+D_4\delta mp^2q)$$
$$\times\frac{X_{pq}}{(m^2-p^2)(n^2-q^2)}+\frac{16}{\pi^3}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}D_5\delta^2(n^2+q^2)\frac{mpY_{pq}}{(m^2-p^2)(n^2-q^2)}=0 \quad (15)$$

After expanding for a given range of values of indices m and n, Eqs. (13-15) give rise to a set of $3mn$ homogeneous algebraic equations in A_{mn}, B_{mn}, and C_{mn}. The stability criterion of the sandwich plate can be formed from the condition that the determinant of the coefficients of A_{mn}, B_{mn}, and C_{mn} should be zero for the nontrivial solution of the unknowns.

Computerization

Equations (14) and (15) are solved for B_{mn} and C_{mn} in terms of A_{mn} and then substituted in Eq. (13). After substitution and simplification, the set of algebraic equations resulting from Eq. (13) can be cast into a matrix form

$$[P]\{A\} = k_x [Q]\{A\} \tag{16}$$

where P and Q are real symmetric matrices and A a column matrix of undetermined coefficients A_{mn} arranged as

$$A = [A_{11}A_{12}\ldots A_{1n}A_{21}A_{22}\ldots A_{2n}\ldots A_{m1}A_{m2}\ldots A_{mn}]^T \tag{17}$$

Equation (16) is in the standard form of a general eigenvalue problem that can be solved by available subroutines. In the present investigation, IMSL subroutines were used with Cyber-174 computer at the Computer Centre, Swiss Federal Institute of Technology, Zurich. For the purpose of numerical evaluation, the first 16 terms in the series of Eq. (9) are retained, i.e., m, $n = 1,2,3,4$.

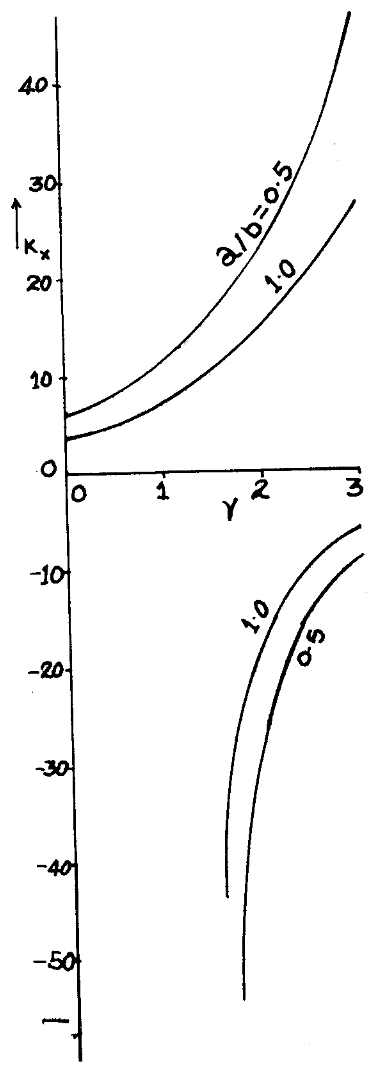

Fig. 5 Variation of k_x with γ ($\theta = 45$ deg, $c/t = 20$).

Results and Discussion

The solution in the present paper is compared with that given in Fig. 4 of Ref. 25. The data used for comparison and taken from this reference are: 1) $a/b = 1$, $a = 225$ mm, $N = 4$; 2) carbon-fiber-faced layers with $E_1 = 229{,}000$ N/mm², $E_2 = 13{,}350$ N/mm², $\mu_{12} = 0.3151$, $G_{12} = 5249$ N/mm², and $t = 0.2$ mm; and 3) aluminum honeycomb core with $G_{xz} = 146$ N/mm², $G_{yz} = 90.4$ N/mm², and $c = 10.0$ mm. The layers are arranged alternately $+\theta$ and $-\theta$ on each face to produce an effectively orthotropic construction, i.e., D_{13} and D_{23} are negligibly small. Using the present solution, the critical loads of uniform compression are evaluated for two cases, $N = 1$ and 4, and presented in Table 1. The stiffness ratios D_1, D_2, D_3, and D_5 are also provided, the first three being independent of number of layers provided the thickness remains the same and D_4 and D_5 varying greatly with number of layers.

Numerical results of the buckling loads are evaluated for an anisotropic sandwich plate with typical glass-fiber-reinforced plastic composite faces and a balsa core under combined longitudinal compression and bending. The principal material properties of face layers and core are

Faces: $E_1 = 24{,}210$ N/mm², $E_2 = 5433$ N/mm²
$\mu_{12} = 0.334$, $G_{12} = 2452$ N/mm²

Balsa core: $G_{xz} = 130$ N/mm², $G_{yz} = 10.0$ N/mm²

With the intention of highlighting the effect of anisotropy on the behavior of the plate, each face is assumed made of a single layer. The response with varying parameters such as aspect ratio a/b, core-to-face thickness ratio c/t, fiber orientation angle θ, and bending load coefficient γ is presented in Figs. 2-5.

The buckling loads of the effectively orthotropic sandwich plates shown in rows 1 and 2 of Table 1 derived, respectively, from Ref. 25 and the present analysis are very close. The latter are larger than the former by 4%. This is justified as the present solution uses Rayleigh-Ritz method, whereas Ref. 25 used a differential equation approach. Comparison of rows 2 and 5 of Table 1 shows that the critical loads of highly anisotropic (i.e., $N = 1$) sandwich plate are less than those of effectively orthotropic (i.e., $N = 4$) sandwich plate, the geometry being same in both cases. At $\theta = 40$ deg, their ratio is about 1.45. One can conclude that the multi-ply-faced effective orthotropic sandwich plates are stronger than single-ply-faced anisotropic sandwich plates.

The buckling coefficient k_x decreases continuously with aspect ratio a/b (Fig. 2) and core-to-face thickness ratio c/t (Fig. 3). The influence of bending load coefficient γ is to increase the buckling coefficient k_x (Figs. 2 and 5). The numerically smallest buckling coefficient k_x is negative (i.e., N_{0x} becomes tensile) at higher values of γ (Fig. 5).

An interesting observation one can make from Fig. 4 is that the longitudinal buckling load N_{0x}, calculated from the graphs and rigidities, continuously decreases with θ for lower aspect ratios, e.g., $a/b = 0.5$, but for aspect ratios greater than or equal to unity, it increases up to $\theta = 40$ deg and then starts decreasing. The maximum longitudinal buckling strength occurs for $a/b = 0.5$ when the fibers are longitudinal and for $a/b = 1$ and 4 when the fibers are oriented at about 40 deg with respect to the longitudinal axis. In the fabrication of anisotropic sandwich plates, one should take into account the aspect ratio in selecting the orientation of reinforcing fibers.

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References

[1]Libove, C. and Batdorf, S. B., "A General Small-Deflection Theory for Flat Sandwich Plates," NACA TN 1526, 1948.

[2]Robinson, J. R., "The Buckling and Bending of Orthotropic Sandwich Panels with all Edges Simply Supported," *Aeronautical Quarterly*, May 1955, pp. 125-148.

[3]Norris, C. B., "Compressive Buckling Curves for Flat Sandwich Panels with Dissimilar Facings," Forest Product Laboratory, Forest Service, U.S. Department of Agriculture, FPL Rept. 1875, Sept. 1960.

[4]Kuenzi, E. W., Norris, C. B., and Jenkinson, P. M., "Buckling Coefficients for Simply Supported and Clamped Flat Rectangular Sandwich Panels under Edgewise Compression," Forest Product Laboratory, Forest Service, U.S. Department of Agriculture, Res. Note FPL 070, Dec. 1964.

[5]Chang, C. C., Ebcioglu, I. K., and Haight, C. H., "General Stability Analysis of Orthotropic Sandwich Panels for Four Different Boundary Conditions," *ZAMM*, Vol. 42, 1962, pp. 373-389.

[6]Nordby, G. M. and Crisman, W. C., "Strength Properties and Relationships Associated with Various Types of Fiber Glass-Reinforced Facing Sandwich Structure," U.S. Army Aviation Materials Laboratories, Fort Eustis, Va., Tech. Rept. 65-15, 1965.

[7]Wempner, G. A. and Baylor, J. L., "General Theory of Sandwich Plates with Dissimilar Facings," *International Journal of Solids and Structures*, Vol. 1, 1965, pp. 157-177.

[8]Stickney, G. H. and Abdulhadi, F., "Flexure Theory of Multi-Layer Orthotropic Circular Sandwich Plates," *Journal of Composite Materials*, Vol. 2, No. 2, 1968, pp. 200-219.

[9]Azar, J. J., "Bending Theory for Multi-Layer Orthotropic Sandwich Plates," *AIAA Journal*, Vol. 6, Nov. 1968, pp. 2166-2169.

[10]Folie, G. M., "Bending of Clamped Orthotropic Sandwich Plates," *Proceedings of American Society of Civil Engineers, Journal of Engineering and Mechanics Div.*, Vol. EM3, June 1970, pp. 243-265.

[11]Azar, J. J. and Johnson, P. C., "Deflection of Orthotropic Sandwich Plates," *AIAA Journal*, Vol. 10, Aug. 1972, pp. 981-982.

[12]Chan, H. C. and Cheung, Y. K., "Static and Dynamic Analysis of Multi-Layered Sandwich Plates," *International Journal of Mechanical Sciences*, Vol. 14, 1972, pp. 399-406.

[13]Chan, H. C. and Foo, O., "Buckling of Multi-Layer Sandwich Plates by the Finite Strip Method," *International Journal of Mechanical Sciences*, Vol. 19, 1977, pp. 447-456.

[14]Ori Ishai and Saggi, "Instability of Sandwich Beams and Columns with FRP Facings," *Proceedings of International Conference on Composite Materials*, Vol. 2, 1975, pp. 247-271.

[15]Webber, J.P.H., Kyriakides, S., and Lee, C. T., "On the Wrinkling of Honeycomb Sandwich Columns with Laminated Cross-Ply Faces," *Aeronautical Quarterly*, June 1976, pp. 264-272.

[16]Gutierrez, A. J. and Webber, J.P.H., "Flexural Wrinkling of Honeycomb Sandwich Beams with Laminated Faces," *International Journal of Solids and Structures*, Vol. 16, 1980, pp. 645-651.

[17]Monforton, G. R. and Ibrahim, I. M., "Analysis of Sandwich Plates with Unbalanced Cross-Ply Faces," *International Journal of Mechanical Sciences*, Vol. 17, 1975, pp. 227-238.

[18]Kulkarni, A. M., Banerjee, J. R., and Sinha, P. K., "Response of Randomly Excited Orthotropic Sandwich Plates," *Journal of Sound and Vibration*, Vol. 41, No. 2, 1975, pp. 197-205.

[19]Schmit, L. A. Jr. and Monforton, G. R., "Finite Deflection Discrete Element Analysis of Sandwich Plates and Cylindrical Shells with Laminated Faces," *AIAA Journal*, Vol. 8, Aug. 1970, pp. 1454-1461.

[20]Monforton, G. R. and Ibrahim, I. M., "Modified Stiffness Formulation of Unbalanced Anisotropic Sandwich Plates," *International Journal of Mechanical Sciences*, Vol. 19, 1977, pp. 335-343.

[21]Monforton, G. R., "Stiffness Matrix for Sandwich Beams with Thick Anisotropic Laminated Faces," *Computers and Structures*, Vol. 10, 1979, pp. 547-551.

[22]Kingsbury, H. B. and Pavicic, N. J., "A Dynamic Theory of Sandwich Plates with Anisotropic Facings," *Fiber Science and Technology*, 1968/69, Vol. 7, pp. 255-272.

[23]Stroud, W. P. Jr. and Kingsbury, H. B., "Study of Stresses and Deformations in Sandwich Plates with Anisotropic Facings," *Journal of Aircraft*, Vol. 9, Dec. 12, 1972, pp. 801-803.

[24]Ibrahim, I. M., Farah, A., and Rizk, M.N.F., "Dynamic Analysis of Unbalanced Anisotropic Sandwich Plates," *Proceedings of American Society of Civil Engineers, Journal of Engineering Mechanics*, Vol. EM 2, 1981, pp. 405-418.

[25]Pearce, T.R.A. and Webber, J.P.H., "Buckling of Sandwich Panels with Laminated Face Plates," *Aeronautical Quarterly*, Vol. 23, May 1972, pp. 148-160.

[26]Pearce, T.R.A. and Webber, J.P.H., "Experimental Buckling Loads of Sandwich Panels with Carbon Fiber Face Plates," *Aeronautical Quarterly*, Vol. 24, Nov. 1973, pp. 295-312.

[27]Jones, R. M., *Mechanics of Composite Materials*, Scripta Rook Co., Washington, D.C., and McGraw-Hill Kogakusha Ltd., Tokyo, 1975, pp. 45-47.